Calculus: The mathematics of change

The following words describe change: increasing, decreasing, growing, shrinking, etc. Change occurs over TIME, so when we describe how a quantity changes, we are talking about the DERIVATIVE of the quantity with respect to TIME.

EXAMPLE 1: Use mathematical notation to describe the following:

a) Sam is growing at the rate of 2 inches/year $\frac{dh}{dt} = +2$ in/yr	b) The radius of a circle is decreasing by 5 feet/second $\frac{dr}{dt} = -5$ ft/s
c) Stephanie's savings account is increasing by a rate of 2 cents/day $\frac{ds}{dt} = +2$ cents/day	d) The volume of a cube is increasing by 10 in^3/sec $\frac{dV}{dt} = +10\ in^3/s$

EXAMPLE 2: A rectangle is $\boxed{\text{10 inches}}$ by $\boxed{\text{6 inches}}$ and its sides are changing. Write formulas for the perimeter and area and how fast each is changing in terms of l and w.

l w

Perimeter: $P = 2l + 2w$	Change in Perimeter: $\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$
Area: $A = l \cdot w$ Product Rule →	Change in Area: $\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$

Scenario 1: The rectangle's length and width are increasing at a rate of 2 in/sec. $\frac{dw}{dt} = \frac{dl}{dt} = 2$

Change in perimeter:

$$\frac{dP}{dt} = 2(2) + 2(2) = 8 \text{ in/s}$$

Change in area:

$$\frac{dA}{dt} = 10(2) + 6(2) = 32 \text{ in}^2/\text{s}$$

Scenario 2: The length and width are decreasing at a rate of 5 in/sec. $\frac{dw}{dt} = \frac{dl}{dt} = -5$

Change in perimeter:

$$\frac{dP}{dt} = 2(-5) + 2(-5) = -20 \text{ in/s}$$

Change in area:

$$\frac{dA}{dt} = 10(-5) + 6(-5) = -80 \text{ in}^2/\text{s}$$

$\frac{dl}{dt} = 3$ $\frac{dw}{dt} = -3$

Scenario 3: The length is increasing at 3 in/sec and the width is decreasing at 3 in/sec.

Change in perimeter:

$\frac{dP}{dt} = 2(3) + 2(-3) = 0$ in/s

Change in area:

$\frac{dA}{dt} = (10)(-3) + (6)(3) = -12$ in²/s

Scenario 4: The length is decreasing at a rate of 2 in/sec and the width is increasing at ½ in/sec.

$\frac{dl}{dt} = -2$ $\frac{dw}{dt} = \frac{1}{2}$

Change in perimeter:

$\frac{dP}{dt} = 2(-2) + 2(\frac{1}{2}) = -3$ in/s

Change in area:

$\frac{dA}{dt} = 10(\frac{1}{2}) + 6(-2) = -7$ in²/s

<u>Steps for solving related rates problems:</u>

1. Make a sketch.
2. Identify all variables: some are CONSTANT and some are CHANGING. Remember, the rate of change is the derivative with respect to time.
3. Write an equation relating the variables. You will probably need to use known formulas.
4. Plug in values for any variable that is a constant. **NEVER plug in a value for a variable that is changing.**
5. Differentiate both sides of your equation with respect to time (*t*). You will use the chain rule and implicit differentiation. Again, make sure you do not plug in any values for variables that are changing before you take the derivative.
6. After you have taken the derivative, plug in all known values for the variables and their rates of change.
7. Solve the equation for the desired quantity. Label your answer using correct units and make sure you answered the question asked.

Example 1: Suppose you were blowing up your balloon at a rate of 10π cubic in/sec. At what rate was the radius increasing when $r = 2$ in?

Example 2: A