Name___________________________

Calculus is defined as the "mathematics of change." The following words describe change: increasing, decreasing, growing, shrinking, etc. Change occurs over TIME, so when we describe how a quantity changes, we are talking about the DERIVATIVE of the quantity with respect to TIME.

EXAMPLE 1: Use mathematical notation to describe the following:

a) Sam is growing at the rate of 2 inches/year	b) The radius of a circle is decreasing by 5 feet/second
c) Stephanie's savings account is increasing by a rate of 2 cents/day	d) The volume of a cube is increasing by 10 in^3/sec

EXAMPLE 2: A rectangle is 10 inches by 6 inches and its sides are changing. Write formulas for the perimeter and area and how fast each is changing in terms of l and w.

Perimeter:	Change in Perimeter:
Area:	Change in Area:

Scenario 1: The rectangle's length and width are increasing at a rate of 2 in/sec.

Change in perimeter: Change in area:

Scenario 2: The length and width are decreasing at a rate of 5 in/sec.

Change in perimeter: Change in area:

Scenario 3: The length is increasing at 3 in/sec and the width is decreasing at 3 in/sec.

Change in perimeter: Change in area:

Scenario 4: The length is decreasing at a rate of 2 in/sec and the width is increasing at ½ in/sec.

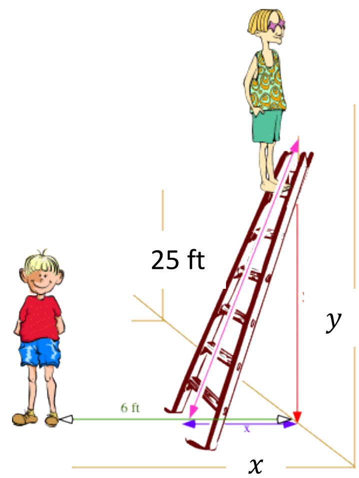
Change in perimeter: Change in area:

<u>Steps for solving related rates problems:</u>

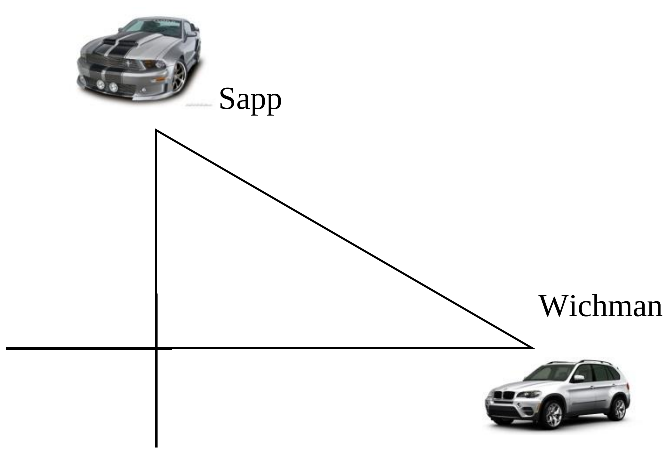
1. Make a sketch.
2. Identify all variables: some are CONSTANT and some are CHANGING. Remember, the rate of change is the derivative with respect to time.
3. Write an equation relating the variables. You will probably need to use known formulas.
4. Plug in values for any variable that is a constant. **NEVER plug in a value for a variable that is changing.**
5. Differentiate both sides of your equation with respect to time (t). You will use the chain rule and implicit differentiation. Again, make sure you do not plug in any values for variables that are changing before you take the derivative.
6. After you have taken the derivative, plug in all known values for the variables and their rates of change.
7. Solve the equation for the desired quantity. Label your answer using correct units and make sure you answered the question asked.

Example 1: A clown is inflating a balloon at a rate of 10π cubic ft/sec. At what rate is the radius increasing when $r = 2$ ft?

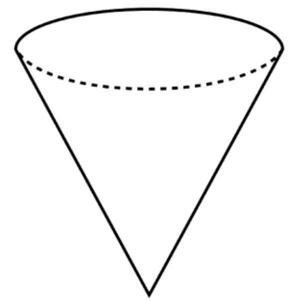
Example 2: A painter is using a ladder 25 feet long which is leaning against a wall. His sadistic friend pulls the ladder away from the bottom of the wall at a rate of 14 ft/sec. At what rate is the top of the ladder moving down the wall when it is 7 feet above the ground?



Example 3: At a given moment, Mrs. Sapp is 30 miles north of an intersection, traveling toward it at 45 mph. At the same time Mr. Wichman is 40 miles east of the intersection, traveling away from it at 35 mph. Is the distance between Mrs. Sapp and Mr. Wichman increasing or decreasing at that moment? At what rate?



Example 4: Water is flowing into a cone-shaped tank at the rate of 5 cubic in/sec. If the cone has altitude of 4 inches and a base radius of 3 inches, how fast is the water level rising when the water is 2 inches deep?



Example 5: A camera is mounted 3000 ft from the space shuttle launching pad. The camera needs to pivot as the shuttle is launched and needs to keep the shuttle in focus. If the shuttle is rising vertically at 800 ft/sec when it is 4000 ft high, how fast is the camera-to-shuttle distance changing?

How fast is the angle of elevation of the camera changing at that moment in time?

PRACTICE:

1. A circle has a radius of 10 inches which is changing. Write formulas for the circumference and area:

Circumference:	Change in Circumference:
Area:	Change in Area:

a) Suppose the radius is increasing at a rate of 2 in/sec. Find:
Change in Circumference: Change in Area when $r = 10$ in:

b) Suppose the diameter is decreasing at a rate of ½ in/sec. Find:
Change in Circumference: Change in Area when $r = 10$ in:

2. A sphere has a radius of 5 feet which is changing. Write formulas for its volume and surface area.

Volume:	Change in Volume:
Surface Area:	Change in Surface Area:

a) Suppose the radius is increasing at a rate of 1/3 ft/min.
Change in Volume when $r = 5$ ft: Change in Surface Area when $r = 5$ ft:

b) Suppose the diameter is decreasing at a rate of 1 ft/min.
Change in Volume when $r = 5$ ft: Change in Surface Area when $r = 5$ ft:

3. Suppose you are filling up a water trough for your horses. The trough is rectangular (6 feet long, 4 feet wide and 3 feet deep.) You are filling the water trough at a rate of $2\ \text{ft}^3/\text{sec}$.

   a) List the values that are constants (not changing).

   b) List the variables that are changing.

   c) Find the rate at which the water is rising.

4. If $A^2 = R^2 + h^2$, find $\frac{dA}{dt}$ when $A = 10$, $R = 8$, $\frac{dR}{dt} = \frac{1}{2}$, and $\frac{dh}{dt} = \frac{1}{3}$

5. Two automobiles start from a point A at the same time. One travels West at 80 miles per hour; the other travels North at 45 miles per hour. How fast is the distance between them increasing 3 hours after they start?

6. A spherical balloon is being inflated at the rate of 12 in^3/sec. How fast is the radius r changing at the moment when $r = 2$ in?

7. Suppose your coach wants you to drink more water before a big game. You bring your water bottle to class. It is cylindrical with a radius of 2 inches. Your classmate thinks it would be funny to poke a hole in the bottom of your water bottle, so the water begins to leak out at a rate of 1 in^3/min. How fast will the water level drop?

8. A ladder 15 feet tall leans against a vertical wall of a house. If the bottom of the ladder is pulled away horizontally from the house at 4 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 9 feet from the wall?

9. A cone (point down) with a height of 10 inches and a radius of 2 inches is being filled with water at the constant rate of 2 in^3/sec. Determine how fast the water surface is rising when the water depth is 6 inches.

10. A streetlight is 15 feet above the sidewalk. A man 6 feet tall walks away from the light at a rate of 5 ft/sec.

    a) Determine a function relating the length of the man's shadow to his distance from the base of the streetlight.

    b) Determine the rate at which the man's shadow is lengthening at the moment he is 20 feet from the base of the light.

11. A construction crew is preparing to build a new house. They are digging out the dirt for a basement and pilling the dirt in a pile. The pile is conical and the dirt is falling onto the pile at a rate of $30 \text{ ft}^3/\text{min}$. You notice that the height of the pile is always $\frac{1}{2}$ of the base diameter. How fast is the height changing when the height of the pile is 12 ft high?

12. A ship is anchored 2 miles off a straight shore, and its searchlight is following a car that is moving along the shore at 40 miles per hour. How fast is the light turning (in radians per hour) when the car is 4 miles from the ship.

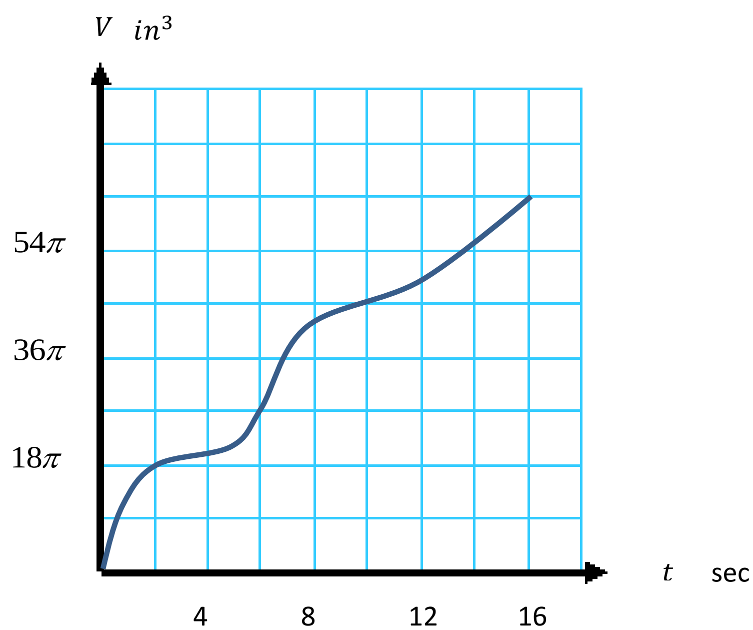
13. The length L of a rectangle is decreasing at the rate of 3 cm/sec while the width W is increasing at the rate of 2 cm/sec. When $L = 12$ and $W = 2$, find the rate of change of

a) the area

b) the perimeter

c) the length of a diagonal.

14. The function V whose graph is sketched below gives the volume of air $V(t)$ that a man has blown into a balloon after t seconds. Approximately how rapidly is the radius changing after 6 seconds? ($V = \frac{4}{3}\pi r^3$)



AP PRACTICE (Multiple Choice)

1. Given that $x^3y + xy^3 = -10$, then $\dfrac{dy}{dx} =$
   a) $\dfrac{3x^2y + y^3}{3xy^2 + x^3}$
   b) $-\dfrac{3x^2y + y^3}{3xy^2 + x^3}$
   c) $-\dfrac{x^2y + y^3}{xy^2 + x^3}$
   d) $3x^2 + 3xy^2$
   e) $-(3x^2 + 3xy^2)$

2. A 26 foot ladder leans against a building so that its bottom moves away from the building at the rate of 3 ft/sec. When the bottom of the ladder is 10 ft from the building, the top is moving down at the rate of r ft/sec, where r is
   a) $46/3$ ft/sec
   b) 3/4 ft/sec
   c) 5/4 ft/sec
   d) 5/2 ft/sec
   e) 4/5 ft/sec

3. CHALLENGE In the triangle below, the hypotenuse has fixed length 5 units, and θ is increasing at a constant rate of $\frac{2}{7}$ radians per min. At what rate is the area of the triangle increasing, in units^2 per minute, when h is 3 units?
   a) 1 unit^2/min
   b) 2 unit^2/min
   c) 3 unit^2/min
   d) 4 unit^2/min
   e) 5 unit^2/min