AP Calculus AB Chapter 6 Packet A

6.1: Rules for Definite Integrals

EXAMPLE 1: Let $F(x) = x\ln x - x$ with derivative $F'(x) = \ln x$. Compute $\int_1^2 \ln x\, dx$

a) Using left and right RSUM with 50 subdivisions.

.379 .393

b) Using the Fundamental Theorem of Calculus.

$\int_1^2 \ln x\, dx = F(2) - F(1)$

$= 2\ln 2 - 2 - [\ln 1 - 1] =$ .386

Total change in $F = F(b) - F(a) = \int_a^b F'(x)\, dx$

Rule	Picture	Example
$\int_a^b c\,f(x)\,dx =$ $c\int_a^b f(x)dx$	f(x)=x; a b; f(x)=2x; a b	$\int_1^3 x\,dx =$ $\frac{1}{2}[2(1+3)] = 4$ $\int_1^3 2x\,dx =$ $\frac{1}{2}[2(2+6)] = 8$
$\int_a^b [f(x) \pm g(x)]\,dx =$ $\int_a^b f(x)dx \pm \int_a^b g(x)dx$	y=2x²; y=3x; 1 2 3; y=1; 1 2 3	$\int_1^3 (2x^2+3x+1)\,dx =$ $\int_1^3 2x^2dx + \int_1^3 3x\,dx + \int_1^3 1\,dx$
$\int_a^b f(x)\,dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ assuming $a < c < b$	f; a c b	If $\int_{-2}^{1} f(x)\,dx = 4$ and $\int_{-2}^{6} f(x)\,dx = -3$ find $\int_1^6 f(x)\,dx$. = -7 $\int_{-2}^{6} f(x)dx = \int_{-2}^{1} f(x)dx + \int_1^6 f(x)dx$

$-3 = 4 + ?$

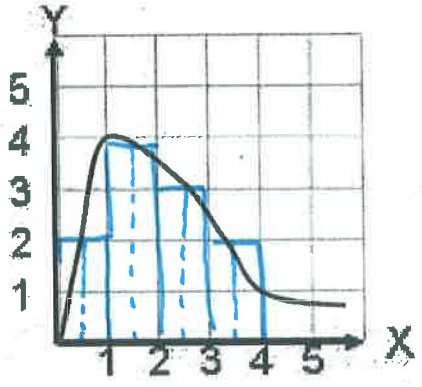
EXAMPLE 2: Apple Inc. estimates the rate of sales of the new iPhone 5S at its Kansas City branch is given by $\frac{dS}{dt} = -3t^2 + 36t$, where t is the number of days after the iPhone was released. What is the total number of iPhones sold in the first five days after the release?

$$\text{Total} = S(5) - S(0) = \int_0^5 (-3t^2 + 36t)\,dt$$

$$= -3\left[\frac{t^3}{3} - 6t^2\right]_0^5$$

$$= -3\left[\frac{5^3}{3} - 6(5^2)\right] = \boxed{325 \text{ phones}}$$

EXAMPLE 3: The rate of extraction of natural gas increases until the easily accessible part is exhausted and then the rate declines. The figure below models the rate $A'(t)$ of natural gas extraction in millions of cubic feet. Estimate the total amount A of natural gas extracted in the first four years.



Let's use midpoint with 4 sub-intervals:

$$A(t) = 2(1) + 3.9(1) + 3(1) + 2(1)$$

$$= \boxed{10.9 \text{ million } ft^3}$$